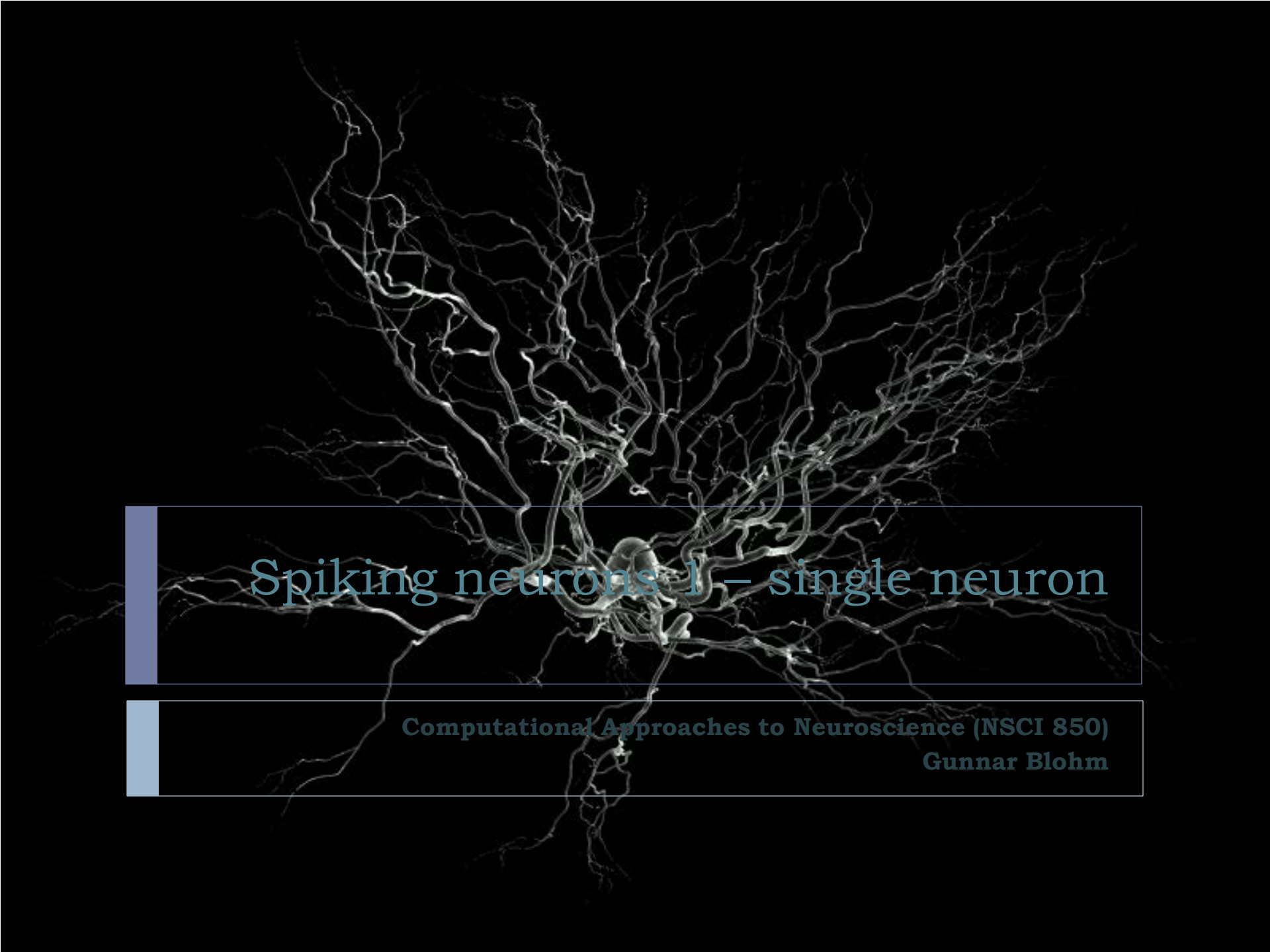


Spiking neurons I – single neuron

Computational Approaches to Neuroscience (NSCI 850)
Gunnar Blohm

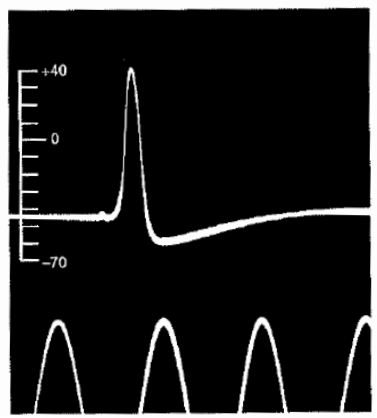
Outline

- (Intro)
- Modelling membrane potentials
  - Ion channels
  - Hodgkin & Huxley
- Modelling synapses
- Other considerations
  - Leaky integrate and fire
  - The Izhiekevich neuron
  - Compartmental models

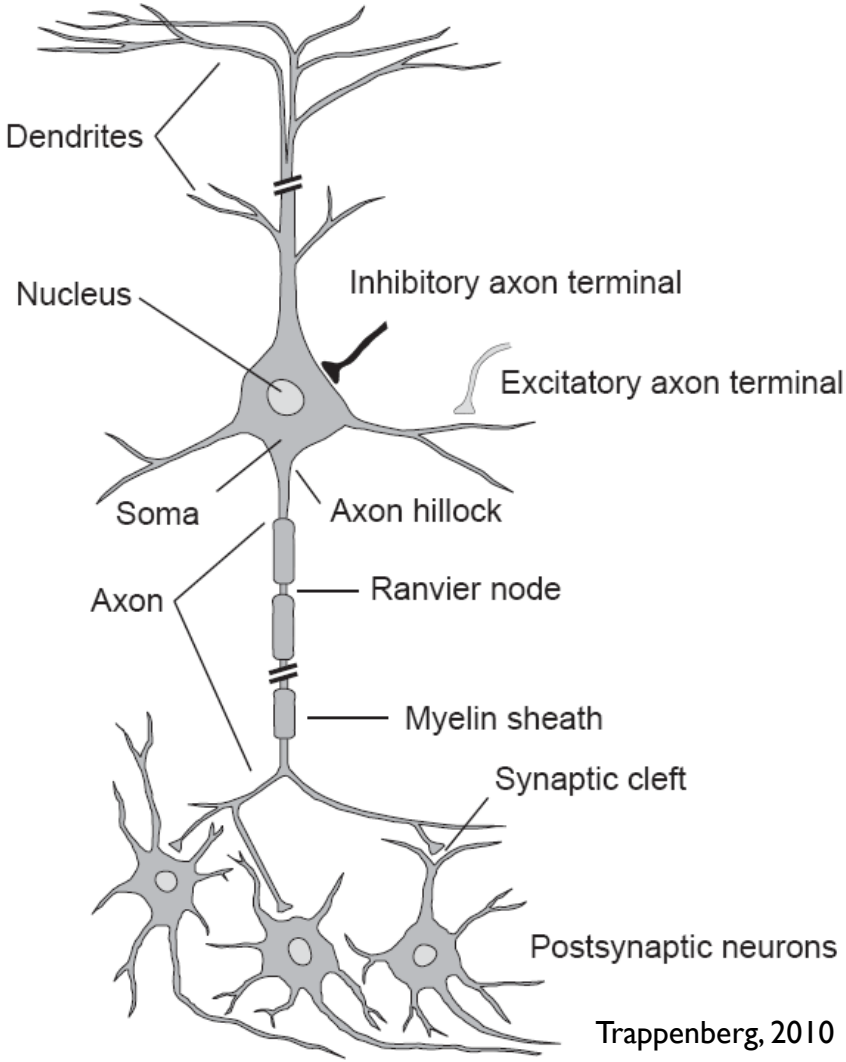
Membrane model

The neuron

- Efferents: typically 10,000
- Pre-synaptic neurons alter state of post-synaptic neurons
- Integration dendric potentials
- Trigger of electric pulse: spike



First action potential: Hodgkin & Huxley, 1939



Trappenberg, 2010

How to build a model neuron…

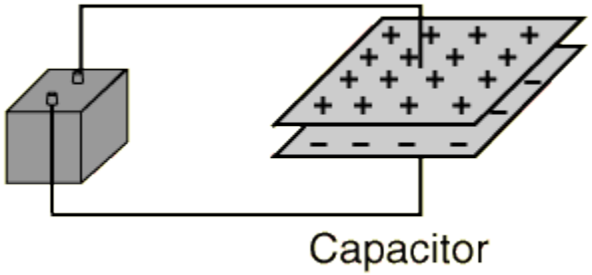
- Intra- versus extra-cellular potential difference (V)
  - Can change over time… → dynamical equations!

- Current *I* is the movement of ions
  - Defined by convention as the direction of net positive charge (cations)
  - Ohm's law: V = I R
  - *Conductance G* is the inverse of *R:* G = I / V

- Membrane: capacitance C=Q/V

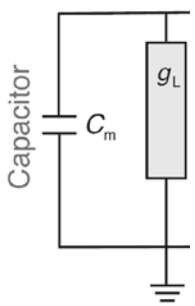
How to build a model neuron…

- Capacitance *C* stores charge *Q* across an *insulator*
  - Q = C*V
  - Q: amount of charge
  - V: potential
  - C: membrane capacitance
    - C = c*S
    - S: membrane surface area
    - c: specific membrane capacitance
  - Effect of a unit of change on membrane potential $V = \frac{Q}{c \cdot S}$
    - Big neuron: small change in V
    - Small neuron: big change in V
- From Q = C*V follows Kirchhoff's law $C \cdot \frac{dV}{dt} = \frac{dQ}{dt} = I$



Example problem

- What happens when a membrane gets charged?
  - Suppose you have only one leaky ion channel (always open)
  - How does the membrane potential evolve over time?



$$C \cdot \frac{dV}{dt} = \frac{dQ}{dt} = I$$

  - I = membrane current = -G*V
  - Solution?
  - Exponential decay!!!

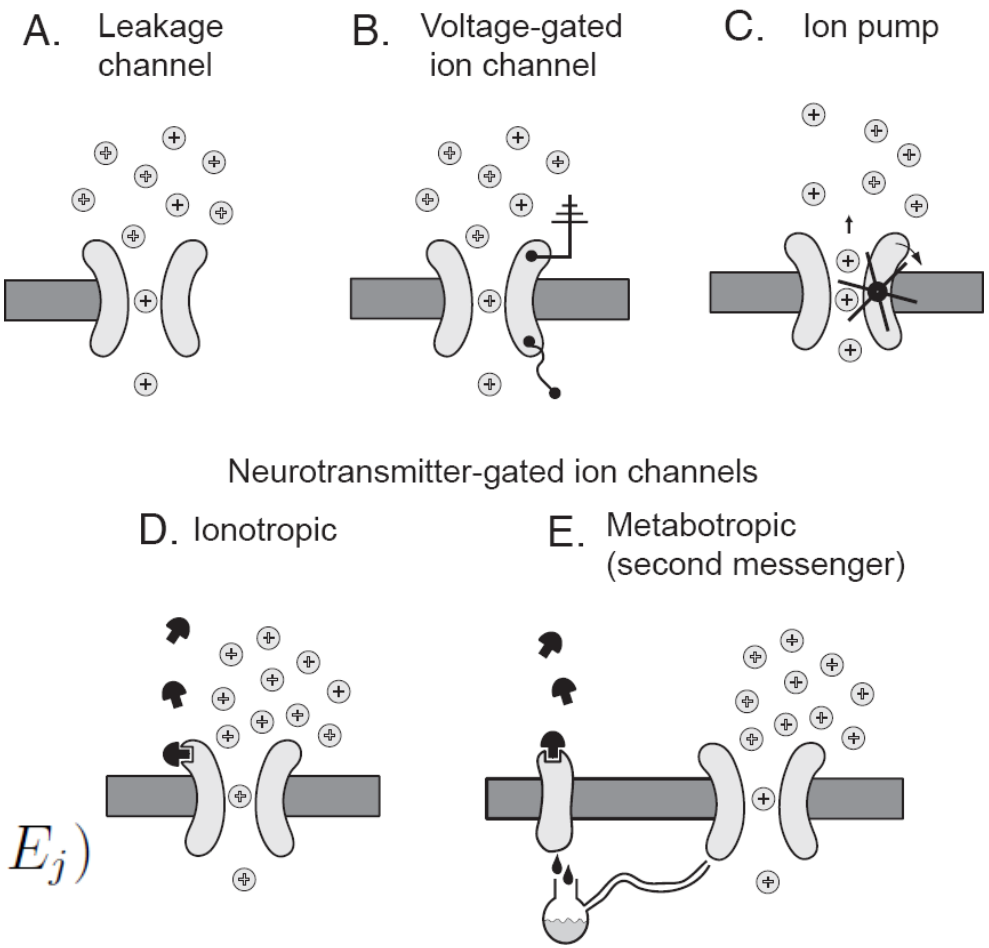
$$V(t) = V_0 \cdot e^{-t/\tau}$$

$$\tau = \frac{C}{G}$$

Ion channels

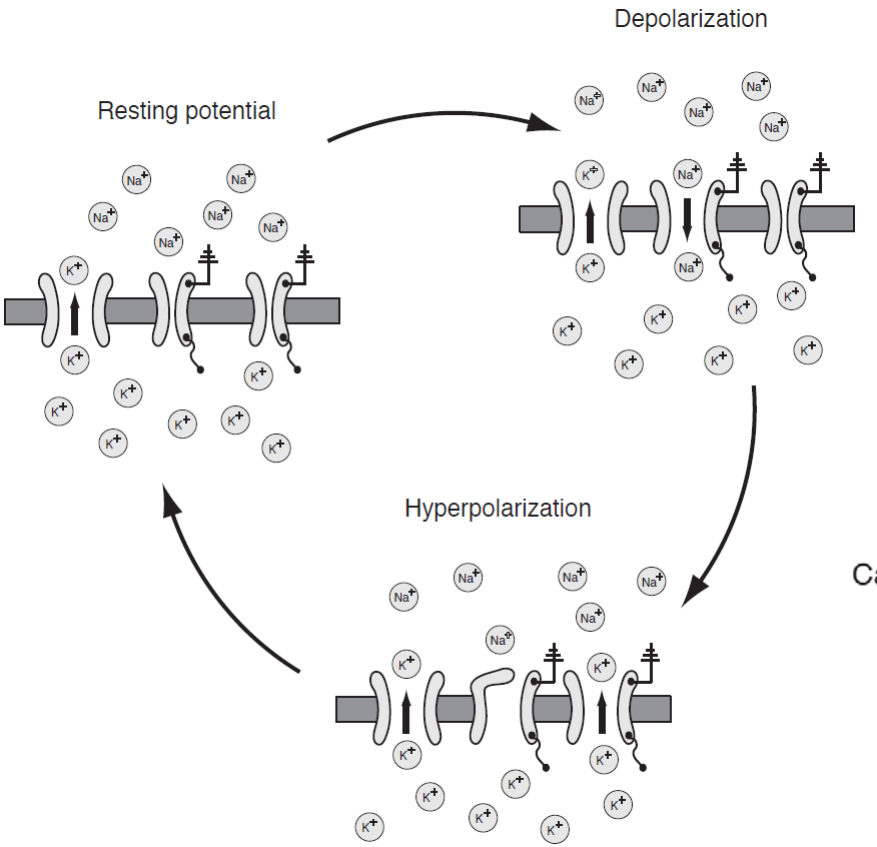
Current flow through ion channels

- Ion channels allow for permeability of membrane
- Nernst equilibrium
  - Electro-chemical balance
  - Equilibrium = reversal potential = E_K = -80mV
  - Same process for Na^+ (but also for Ca^{2+} and Cl^-)
  - Result: resting potential of neuron: V_{rest} = -65mV
- Net current: $I_m = \sum_j g_j(V - E_j)$



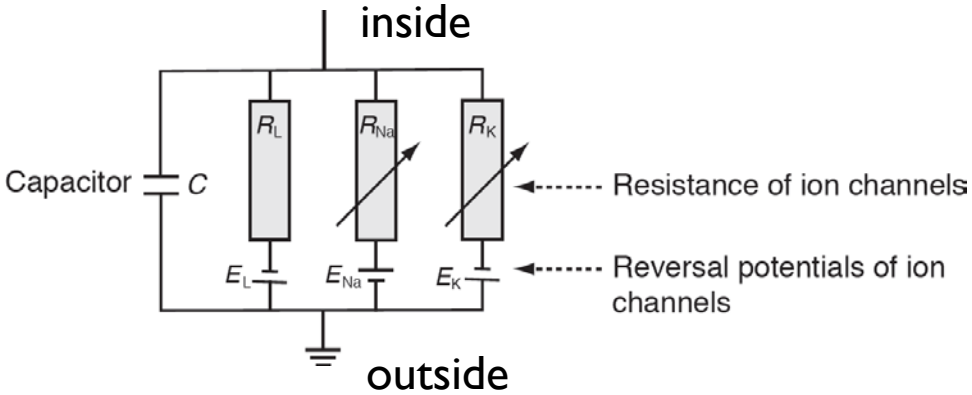
Trappenberg, 2010

Hodgkin-Huxley neuron model

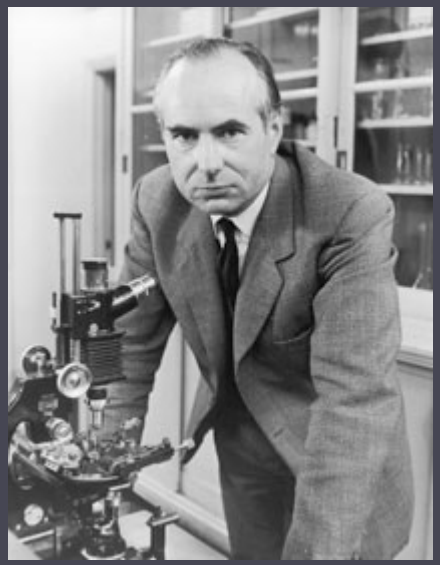


Equivalent electrical circuit

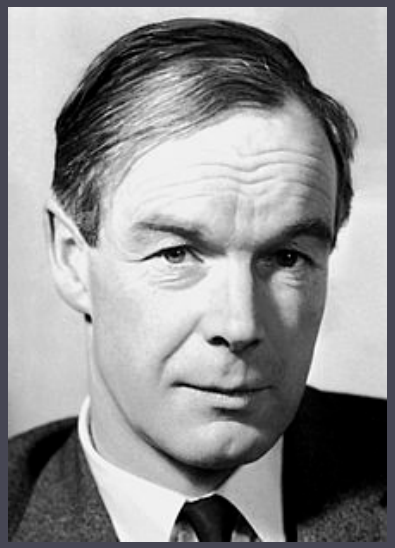
Conductors / resistors representing ion channels
Capacitor representing charge stored across membrane
Batteries representing concentration gradients of relevant ions



Trappenberg, Fundamentals of Computational Neuroscience, 2nd ed.



Sir Alan Hodgkin (1014-1998)



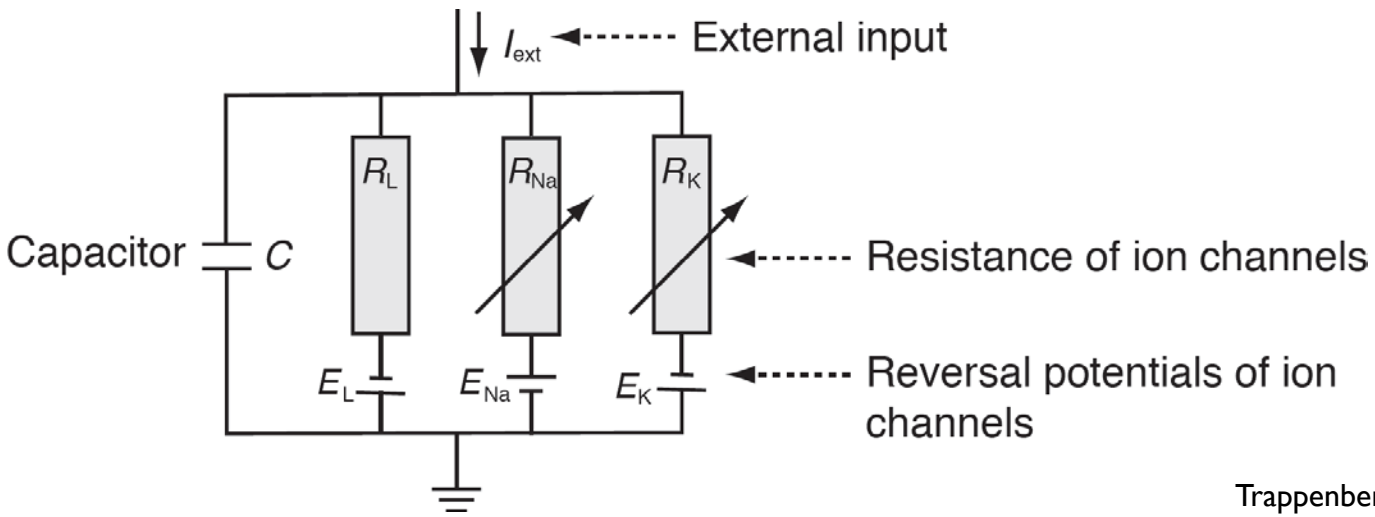
Sir Andrew Huxley (1917-2012)

Hodgkin-Huxley neuron model

Nobel prize in Medicine (1963)

Hodgkin-Huxley equations

- Hodgkin and Huxley introduced empirically 3 variables that describe the voltage-dependence of the ion channel conductance
  - n = n(V,t): activation of K channels
  - m = m(V,t): activation of Na channels
  - h = h(V,t): inactivation of Na channels

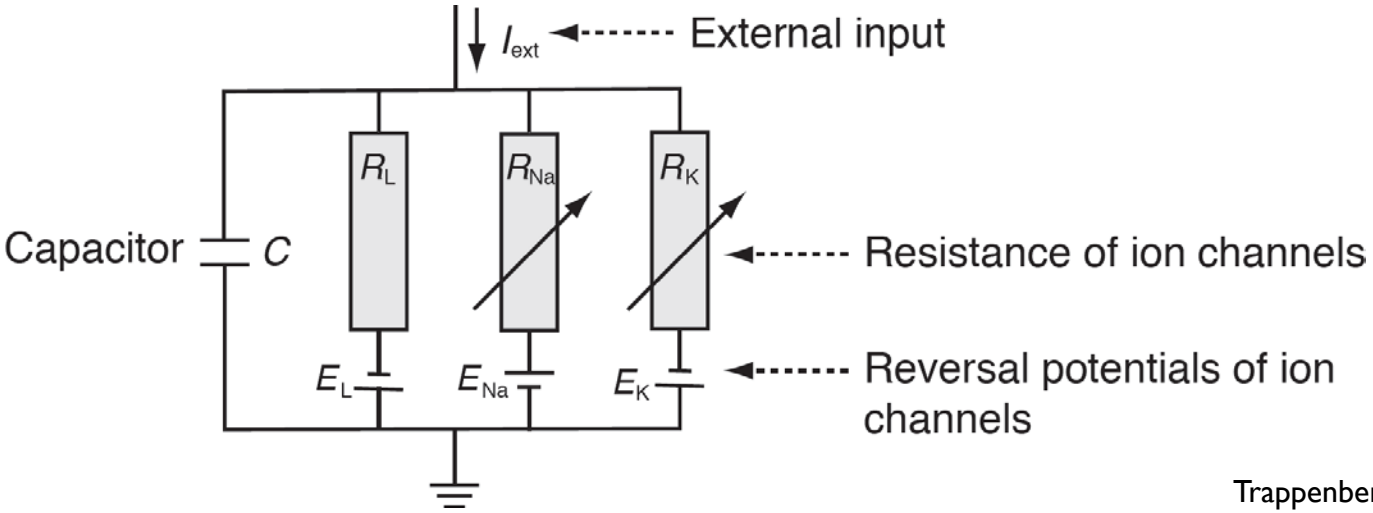


Trappenberg, 2010

Hodgkin-Huxley equations

Giant squid axon model

$$C \cdot \frac{dV}{dt} = -g_K n^4 \cdot (V - E_K) - g_{Na} m^3 h \cdot (V - E_{Na}) - g_L \cdot (V - E_L) + I(t)$$



Trappenberg, 2010

Hodgkin-Huxley equations

$$C \cdot \frac{dV}{dt} = -g_K n^4 \cdot (V - E_K) - g_{Na} m^3 h \cdot (V - E_{Na}) - g_L \cdot (V - E_L) + I(t)$$

$$\tau_n(V)\frac{dn}{dt} = -[n - n_0(t)]$$ K+ activation

$$\tau_m(V)\frac{dm}{dt} = -[m - m_0(t)]$$ Na+ activation

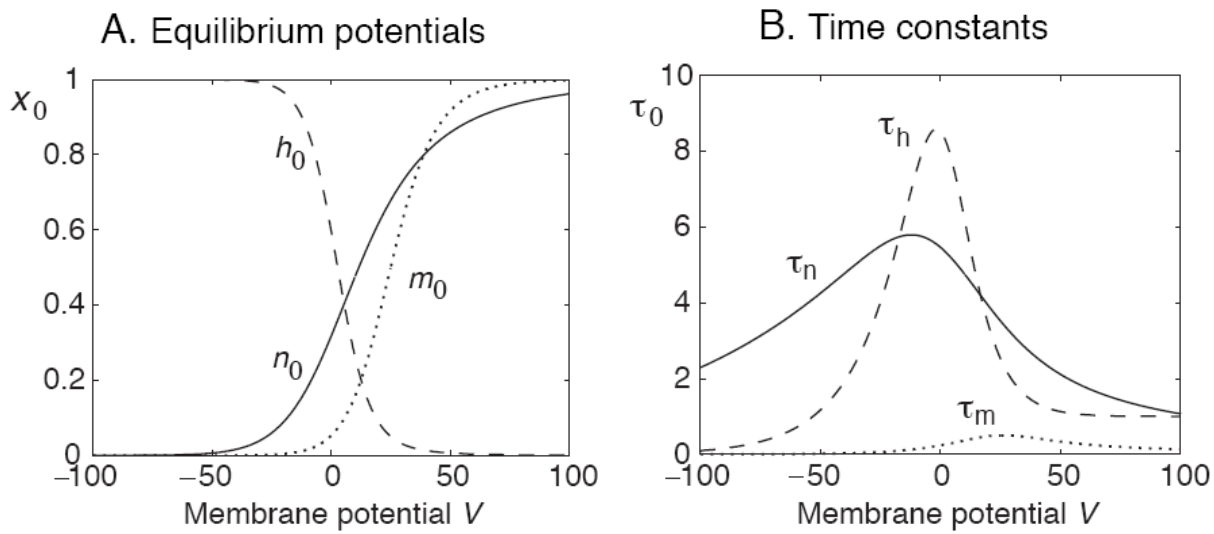
$$\tau_h(V)\frac{dh}{dt} = -[h - h_0(t)]$$

Na+ *de*activation

Conventions

Membrane current is defined as positive outward

Electrode currents are defined as positive inward

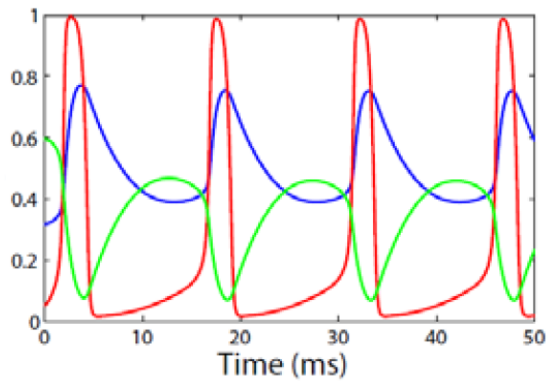


Trappenberg, 2010

Simulations

- Numerical integration, e.g. using Euler method

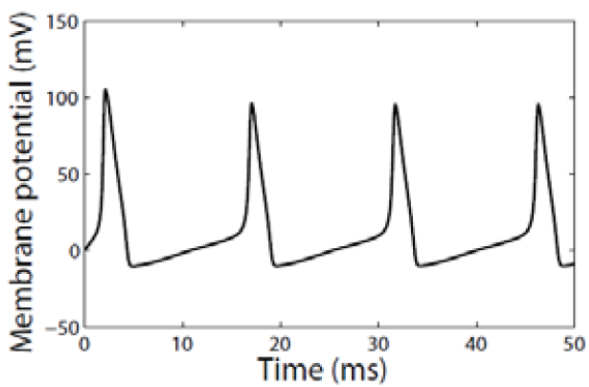
$$x(t+\Delta t) = x(t) + \Delta t \cdot \dot{x}(t)$$



n (K+ activation)

m (Na+ activation)

h (Na+ deactivation)



e.g. 10 mA
current injection

… or Matlab's ODE solvers

Exercises…

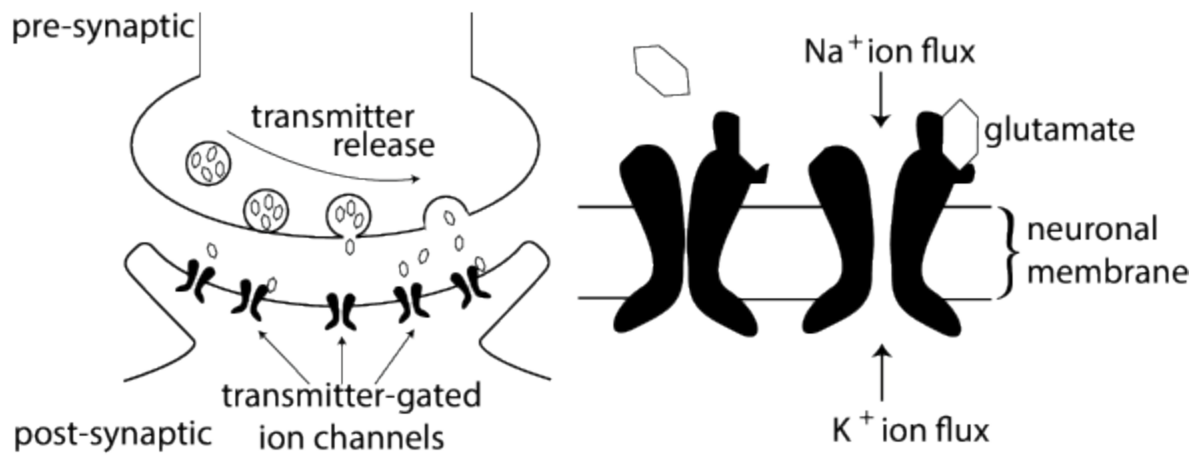
Download "HodgkinHuxley.m"

1. How do spiking frequency and spike amplitude depend on the stimulation current?
2. Introduce a second current step 100ms after the first one and analyze the spiking behaviour. How does spiking depend on the current step / initial current? What happens when using a staircase stimulation? Also try negative currents!
3. After a spike, the membrane potential enters the so-called refractory period. Explore the effect of refractoriness on triggering a subsequent spike by introducing a second current step right after the first spike.

Modelling synapses

Modelling synapses

- General equation: $$I_j = p\ \alpha_j\ g_j(V - E_j)\ W\ \eta$$
  - j: AMPA, GABA, NMDA
  - W: connectivity strength (weight)
  - E: equilibrium potential (AMPA, NMDA: 0mV; GABA: -70mV)
  - η: voltage dependence of NMDA receptors
  - p: proportion of channels opening
  - α: synaptic gating variable
  - g: conductivity



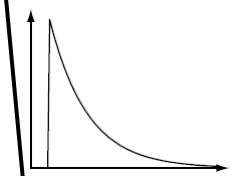
http://neuronaldynamics.epfl.ch/index.html

Modelling synapses

Input spike train

For AMPA, GABA

$$\frac{d\alpha_j}{dt} = -\frac{\alpha_j}{\tau_j} + \delta(t - t_f)$$



For NMDA

$$\frac{d\alpha_{NMDA}}{dt} = -\frac{\alpha_{NMDA}}{\tau_{NMDA}} + \gamma\ x(1 - \alpha_{NMDA})$$

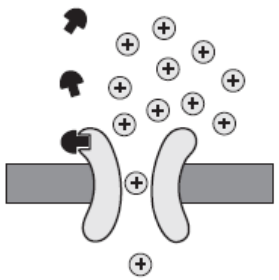
$$\frac{dx}{dt} = -\frac{x}{\tau_x} + \delta(t - t_f)$$

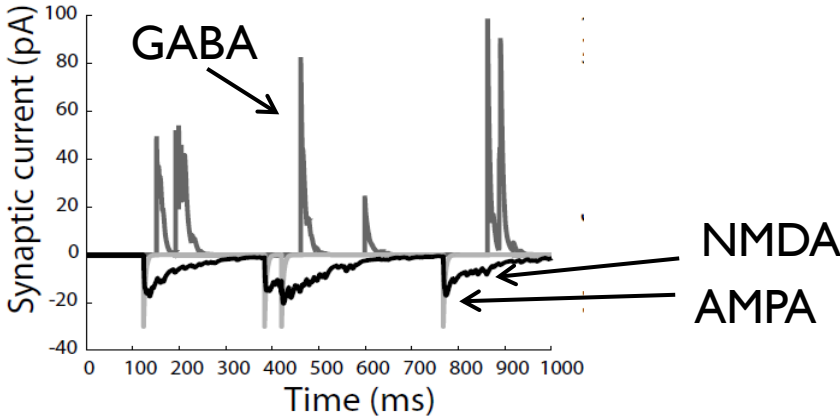
voltage dependence of NMDA receptors

$$\eta = 1/(1 + Mg\ \cdot exp(-0.062.V)/3.57)$$

Jahr and Stevens, *J Neurosci*, 1990

γ = 0.5kHz
τ_x = 2ms
τ_{NMDA} = 100ms
τ_{GABA} = 10ms
τ_{AMPA} = 4ms
Mg = 1mM



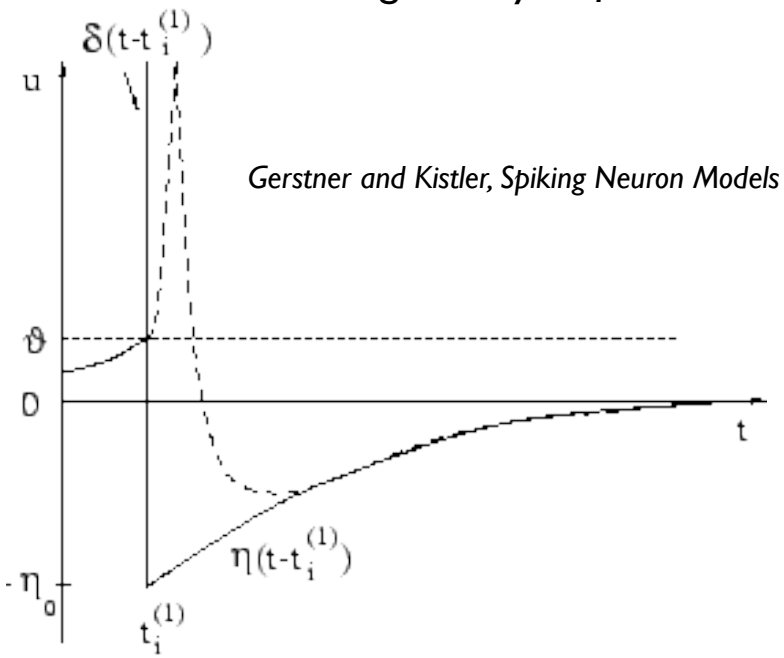


Leaky integrate and fire

Leaky integrate-and-fire neuron

If all spikes from a given neuron are the same, do we need to model the spike?

Timing conveys information, not spikes themselves



Gerstner and Kistler, Spiking Neuron Models

$$\eta \quad \begin{cases} 1/\Delta t & \text{for } 0 < t - t_i^{(f)} < \Delta t \\ -\eta_0 \exp\left(-\frac{t-t_i^{(f)}}{\tau}\right) & \text{for } \Delta t < t - t_i^{(f)} \end{cases}$$

Sub-threshold dynamics only

Note: AP generation mechanism **can** be modulated, so we can't always defend leaving out the spike …

Leaky integrate-and-fire neuron

$$\tau_m \frac{dV}{dt} = -V + IR$$

… does not consider equilibrium potential

Reminder of conventions

Membrane current defined as negative inward

Electrode current defined as positive inward

$$\tau_m \frac{dV}{dt} = E_L - V + RI_e$$

Leaky integrate-and-fire neuron

$$\tau_m \frac{dV}{dt} = E_L - V + RI_e$$

Subthreshold dynamic

$$t^f : V(t^f) = V_{thr}$$

Spike threshold

$$\lim_{t \to t^f \; ; \; t > t^f} V(t) = V_{res}$$

Reset

$$V(t^f + \Delta^{abs}) = V_{res}$$

Absolute refractory period

Leaky integrate-and-fire neuron

$$\tau_m \frac{dV}{dt} = E_L - V + RI_e$$

LIF subthreshold

$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$

Euler integration

Sample code

```
V = [ ]; F = [ ];
dt = 1; % timestep (ms)
for t = 1:100 % 100 ms running time
    v = v + dt/tau_m * (E_L - v + R * Ie) .* (t - t_f > tau_ref);
    f = v > v_th; % Boolean
    % Reset fired to v_res
    v = v + f .* (-v + v_res);
    t_f = f * t + (1 - f) .* t_f; % For abs. refrac. period
    % Record firing
    if f
        F=[F; t];
    end
    V = [V; v];
end
```

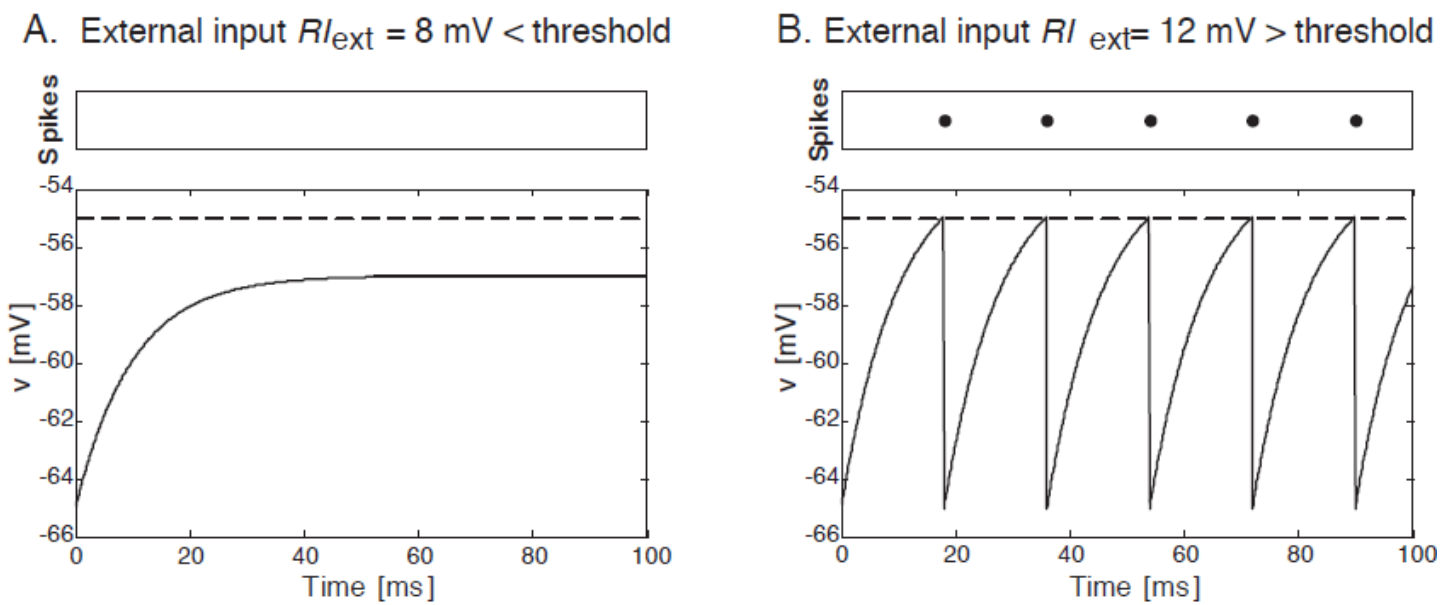
Leaky integrate-and-fire neuron

$$\tau_m \frac{dV}{dt} = E_L - V + RI_e$$

LIF subthreshold

$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$

Euler integration



Trappenberg, Fundamentals of Computational Neuroscience, 2nd ed.

Izhiekevich neurons

Izhikevich neurons

HH neuron : fairly thorough characterization of neural dynamics
LIF neuron : computationally cheap

Is there anything in between ?

e.g. the Izhikevich neuron model
Very flexible, can simulate a variety of spiking behaviour, and computationally cheap

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

$$\text{if } v \geq 30 \text{ mV, then} \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

a : timescale of recovery variable u
b : sensitivity of u to subthreshold fluctuations of v ; how strongly u and v are coupled
c : after-spike reset of v
d : after-spike recovery of u

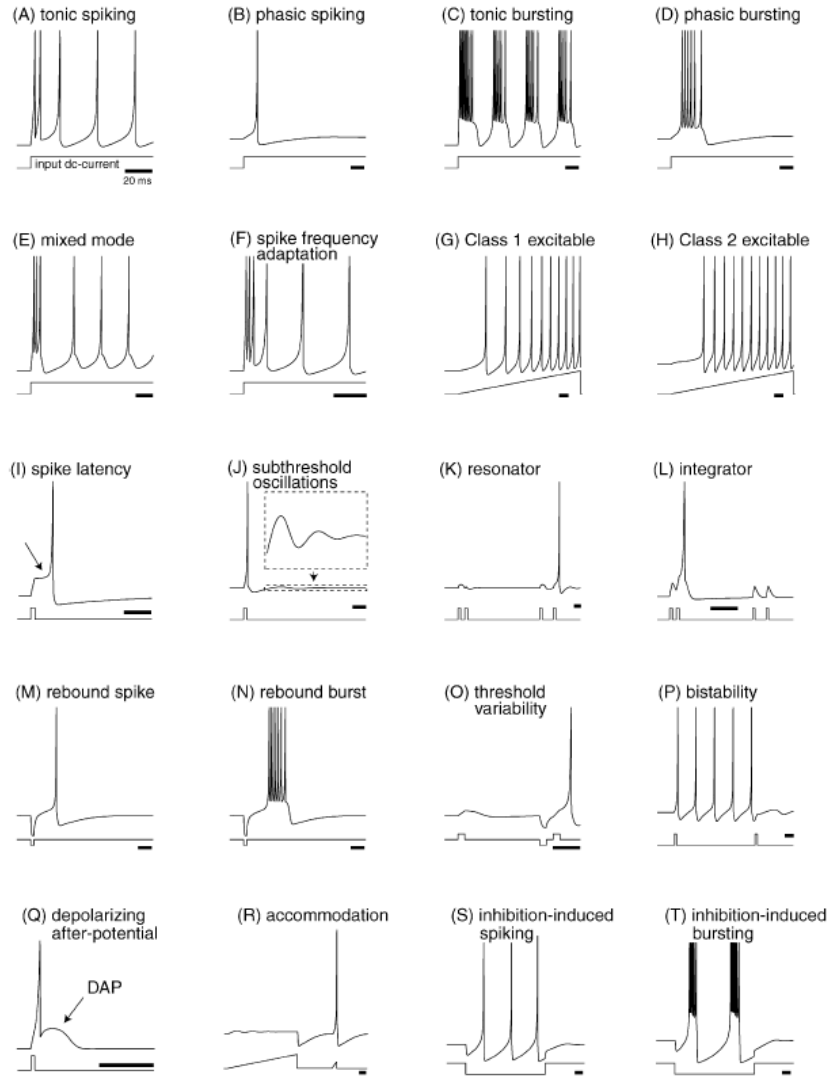
E. Izhikevich, IEEE Trans. in Neural Networks, 16:2003

Izhikevich neurons

- Simplified model for fast simulations

$$\frac{dv(t)}{dt} = 0.04v^2 + 5v + 140 - u + I(t)$$
$$\frac{du(t)}{dt} = a(bv - u)$$

$$v(v > 30) = c \text{ and } u(v > 30) = u - d$$



Izhikevich, IEEE Transactions on Neural Networks (2004)

Compartmental models

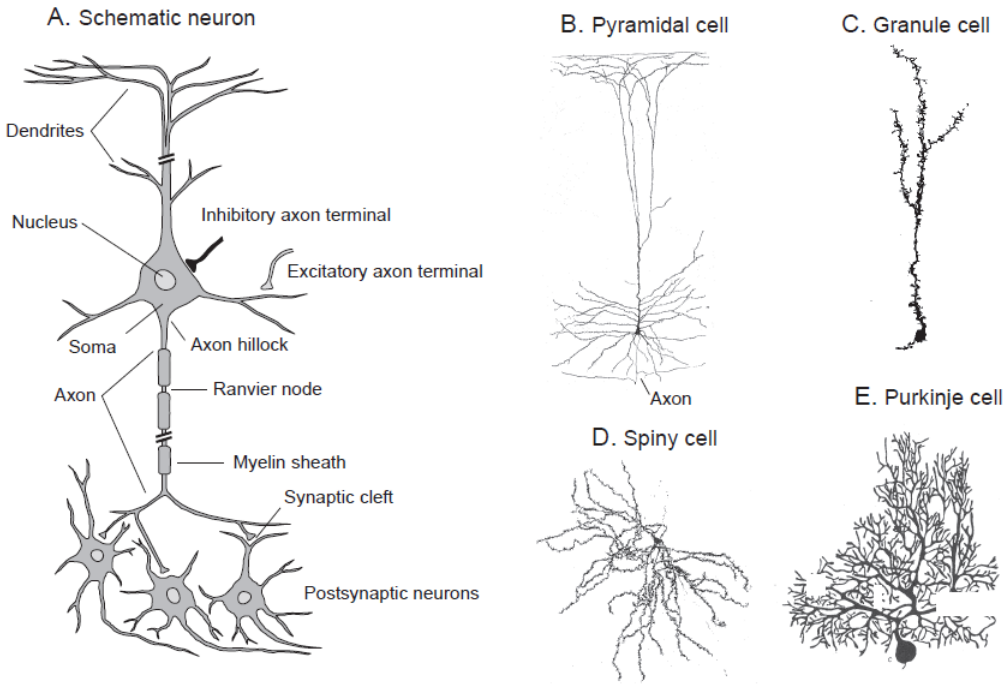
Discussion

What about neuronal shape ?

i.e. dendritic computation

We have assumed uniform resistance, a huge assumption

When is the assumption defensible?



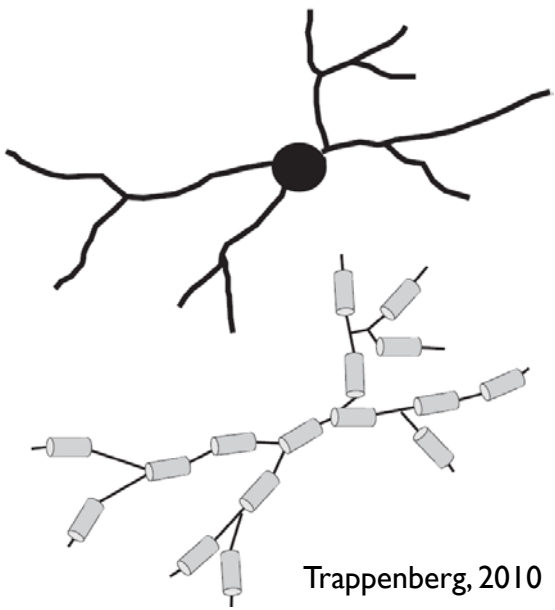
Trappenberg, Fundamentals of Computational Neuroscience, 2nd ed.

Compartmental models

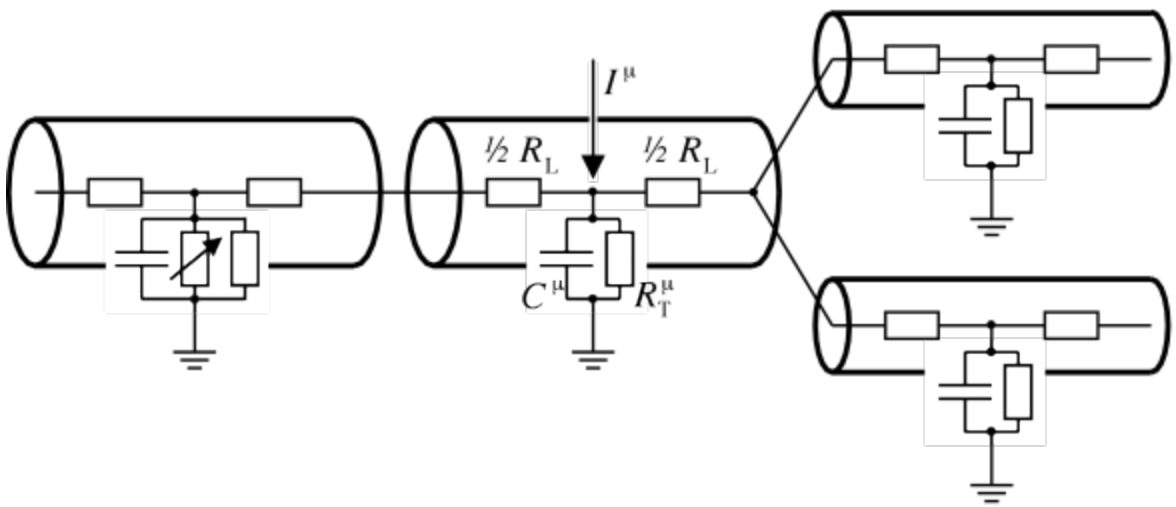
- Allows to include neuronal morphologie
- Cable equation

$$\frac{r_m}{r_l} \cdot \frac{\partial^2 V}{\partial x^2} = c_m r_m \frac{\partial V}{\partial t} + V$$

  - r_m, r_l: membrane, segment resistivity
  - c_m: membrane capacitance



Trappenberg, 2010



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